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These are the only cases in which the group G is transitive. Therefore we conclude that the necessary and sufficient conditions for the irreducibility of (1) are, if $a \neq 0$,

I) $\sqrt{(2-b+\frac{1}{4}a^2)}$ irrational, and $\sqrt{[(1+\frac{1}{2}b)^2-a^2]}$ irrational.

II) $\sqrt{(2-b+\frac{1}{4}a^2)}$ irrational, $\sqrt{[(1+\frac{1}{2}b)^2-a^2}$ rational, and $\sqrt{\{\frac{1}{4}a^2-(1+\frac{1}{2}b) \pm \sqrt{[(1+\frac{1}{2}b)^2-a^2]}\}}$ irrational.

III) When $a=0$, $\sqrt{(-2-b)}$, $\sqrt{(2-b)}$, $\sqrt{(b^2-4)}$ all irrational. When these conditions are not fulfilled the equation is reducible and its group may be any of the intransitive subgroups of G_8. By examining the values of the set of functions (5)-(12) for any particular case the groups G can be determined. The following cases arise:

1) ϕ rational, ψ irrational. Hence, $G=H_4$, H_2, or H'_2, dependent upon rational or irrational nature of x_0, x_1. Since ψ is irrational, x_0 and x_1 are not both rational. Therefore $G \neq G_1$.

2) ϕ rational, ψ rational. Hence, $G=G_2 \equiv [1;\ (\alpha_0\beta_0)(\alpha_1\beta_1)]$, the greatest common subgroup of G_4, H_4 and C_4, or if all remaining functions are rational, then $G=G_1$.

3) ϕ irrational, ψ rational. Hence, $G=G'_2$ or G''_2, according as ϕ' or ϕ'' is rational. The case ϕ', ϕ'' both irrational has been included in the previous discussion.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

GEOMETRY.

322. Proposed by FRANK LOXLEY GRIFFIN, S. M., Ph. D., Instructor in Mathematics, Williams College.

Find all surfaces such that the normal lengths intercepted by the three coordinate planes are in constant ratios for all points.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

$$\left.\begin{array}{l} n-y+\dfrac{dz}{dy}(s-z)=0, \\[2ex] m-x+\dfrac{dz}{dx}(s-z)=0, \end{array}\right\} \text{ is the equation to the normal, where } (x,\ y,\ z)$$

is the point on the surface.

$(x+zdz/dx,\ y+zdz/dy,\ 0)$, where the normal meets the xy plane.

$(x-ydy/dx,\ 0,\ ydy/dz+z)$, where the normal meets the xz plane.

$(0,\ y-xdx/dy,\ xdx/dz+z)$, where the normal meets the yz plane.

Then

$$\pm\frac{x\sqrt{[1+(dx/dy)^2+(dx/dz)^2]}}{y\sqrt{[1+(dy/dx)^2+(dy/dz)^2]}}=a=\pm\frac{xdx}{ydy}\ldots(1).$$

$$\pm\frac{x\sqrt{[1+(dx/dy)^2+(dx/dz)^2]}}{z\sqrt{[1+(dz/dx)^2+(dz/dy)^2]}}=b=\pm\frac{xdx}{zdz}\ldots(2).$$

$$\pm\frac{y\sqrt{[1+(dy/dx)^2+(dy/dz)^2]}}{z\sqrt{[1+(dz/dx)^2+(dz/dy)^2]}}=c=\pm\frac{ydy}{zdz}\ldots(3).$$

$\therefore aydy=xdx$, $bzdz=xdx$, $czdz=ydy$, $aydy+bzdz+czdz=2xdx+ydy$.
$\therefore 2x^2=(b+c)z^2+(a-1)y^2+D$.
$\therefore x^2=Ay^2+Bz^2+D$ are the surfaces satisfying the conditions.

The equation could be put in the form $x^2=ay^2-acz^2+D=0$, where $ac=-b$, as is the case from (1), (2), and (3).

323. Proposed by W. J. GREENSTREET, Marling School, Stroud, England.

S, S' are the foci of two co-vertical parabolas A and B, the axes of which are at right angles. Draw the circle K on SS' as diameter. K is cut in D and E by a straight line parallel to the axis of A such that S' lies midway between it and that axis. Show that the lines $S'D$, $S'E$ are parallel to the two tangents to A which are normals to B.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

The problem as stated is not true.

Let $y^2=4ax$, $x^2=4by$, be the parabolas A, B; $y=mx+a/m$=tangent to A; $y-y'=m(x-x')$ the normal to B.

Since $x'^2=4by'$, $m=-2b/x'$ or $x'=-2b/m$, $y'=x'^2/4b=b/m^2$.

$\therefore y=mx+2b+b/m^2$=normal to B.

But $y=mx+a/m$ and $y=mx+2b+b/m^2$ are the same line.

$\therefore a/m=2b+b/m^2$.

$\therefore m=\frac{1}{4b}[a\pm\sqrt{(a^2-8b^2)}]$, which is true for $a\geqq 2\sqrt{2}\,b$.

$x^2-ax+y^2-by=0$ is equation to K, $y=2b$=line parallel to axis of A.

$\therefore x^2-ax+2b^2=0$ or $x=\frac{1}{2}[a\pm\sqrt{(a^2-8b^2)}]$.

$\therefore y=\frac{2bx}{a\pm\sqrt{(a^2-8b^2)}}+b$ are the equations to $S'D$, $S'E$.

$\therefore$ If m were twice as great, $S'D$, $S'E$ would be perpendicular to the two tangents to A which are normals to B.

The lines through S' parallel to the two tangents are given by the equation

$$y=\frac{1}{4b}[a\pm\sqrt{(a^2-8b^2)}]x+b.$$

This line intersects K in the points